A Remark on the Constancy of the Velocity of Light[1]

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I Introduction

In his famous paper of 1905 Einstein postulated that the velocity of light be constant in all inertial systems. Measurements with increasing accuracy confirmed the justification of this conjecture, so that today we have in fact replaced the normal meter by the cesium second and nine numbers. A physicist measuring the velocity of light, who would come up with a result different from the nine legal numbers, would just have used an illegal system of units.

In his General Theory of Relativity (GRT) Einstein conceded that the velocity of light may depend on the gravitational potential which would explain the deflection of light passing heavy masses. In order to verify any variation of c experimentally, one would need to measure the velocity of light at different gravitational potential. This is, however, no longer possible having abolished the normal meter. It is, therefore, now custom to postulate also in GRT the constancy of c and explain the observed deflection and the longer duration of the passage of light near gravitational centers by a distorted metric of space.

There are, however, experiments carried out on earth which are much easier to interpret when we follow Einstein's original conjecture, namely that the velocity of light depends on the gravitational potential. Curiously enough, these experiments are commonly taken as a confirmation of GRT, but this is in fact only true when we allow for a variation of the velocity of light. If not, a violation of the energy principle would be the consequence.

In this note we discuss the famous experiment by Pound and Rebka of 1960 who used the Mössbauer effect to measure the "apparent weight of photons". We compare it with the "Maryland experiment" of Alley proving that atomic clocks run faster with increasing distance from the gravitational center. Both experiments are frequently said to confirm in an "equivalent" way a simple formula derived in GRT, but, as already hinted by Pound and Rebka, this is actually not true. We include in our comparison a simple gedanken experiment which is based on the conservation of energy and we come to the conclusion that the real experiments may be reconciled when we allow for a variation of the velocity of light.

II Two experiments checking on certain predictions of GRT

As soon as the Mössbauer effect offered a relative accuracy of 10^{-15} Pound and Rebka used it immediately to check on two predictions of GRT: 1) Clocks run faster with increasing distance from a gravitational center. 2) The energy of photons increases when they "fall" in a gravitational field. The first prediction is, of course, not generally true. If the clock is a pendulum-clock, it swings definitely at a reduced frequency on top of a mountain. If

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the clock is the rotating earth itself, it has obviously the same frequency on the mountain and in the valley. None of these clocks, however, offers the precision to detect the faint effect predicted by GRT:

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\Phi}{c^2} \tag{1}$$

which is accessible by atomic clocks using the equivalence:

$$h\,\nu = E_2 - E_1 \tag{2}$$

If one stipulates that the energy difference of two atomic levels depends on the gravitational potential in a way to yield (1), atomic clocks should run faster at higher elevation. On the other hand, if one ascribes an effective mass to a photon according to the relationship:

$$h\,\nu = m_{eff}\,c^2 \tag{3}$$

”falling” photons should gain energy in the gravitational field and the increase in frequency would be precisely that given by (1). In the Pound-Rebka experiment the photon emitting nuclei are at an elevated height **and** the photons fall to the bottom of the tower where the absorber is placed. One would, therefore, expect to observe twice the effect given by (1). As only the single effect was measured, Pound and Rebka quote two “schools of thought”: School I says that clocks run faster on the top of the tower, but the photons do not change their frequency on the way down. School II claims that the frequency of clocks is independent of their elevation, but the falling photons increase their energy and thereby their frequency. Although the authors tend towards school II by entitling their paper “apparent weight of photons”, they leave the question open, also in the second publication by Pound which confirms (1) with higher accuracy. In modern seminars students are told that the contradicting views of school I and II are “equivalent” which “solves” the problem terminologically.

Since Alley’s experiment of 1976 it is possible to distinguish between the two schools. By flying atomic clocks at a height of 10 km at low speed to minimize the velocity effect predicted by SRT, it was possible to confirm equation (1) by a direct comparison of the clocks after landing, without dropping photons from the aircraft to the ground station. Taking together the results of Pound and Alley it is now clear that photons do not change their frequency when approaching or escaping from a gravitational center. School I is confirmed by experiment. Pound and Rebka measured the single effect, because the emitting nuclei produced higher energetic photons on the top of the tower.

This conclusion raises, of course, a problem. The deflection of star light passing “Einstein-lenses”, of which the first one known was our sun, seems to favor school II, since photons are apparently attracted by a gravitational center. In the next Section we consider a simple gedanken experiment based on the conservation of energy and find that the experiments of Alley and Pound can be reconciled with Einstein’s original conjecture assuming a variation of the velocity of light.

III A gedanken experiment

Consider a closed box containing a drop of water and sitting on a scale. If the box is heated, the water vaporizes, but the reading of the scale is not changed. We may draw this conclusion without going into details by applying the energy principle: If the box became lighter by evaporating the water, we could lift it to some height, condense the water there, and let the box fall to the original level. We would gain energy from the heavier falling box by just performing a cycle in a conservative field. It is, however, obvious that the free molecules lose energy on their way up to the lid and gain energy when falling to the bottom so that upon reflection the average force on the bottom is greater than that on the lid. Doing the exact calculation one finds that the weight as measured by the scale is the same regardless whether the water consists of free molecules or of a condensed drop.

Replacing now the heated drop of water by a flashlight switched on inside a box with reflecting walls we have the same expectation: Although part of the rest mass of the box is converted into radiation energy in accordance with (3), we should still be able to measure the weight of the radiation field, otherwise a violation of the energy principle would occur. If, however, the falling photons, in contrast to atoms, do not change their energy

$$E = h\,\nu \tag{4}$$

since their frequency stays constant as concluded from Pound's and Alley's experiments, it seems that the box with the flashlight on is indeed lighter than with the light put out. The paradox is resolved when we consider the momentum of the photons:

$$p = \frac{h\,\nu}{c} \tag{5}$$

The total force on bottom and lid of the box results from the momentum exchange with the walls upon reflection of the photons. If their frequency does not depend on height, it must be the velocity of light which depends on the gravitational potential such that the radiation pressure is increased at the bottom according to (5). In order to comply with the above energy argument, the "weight" of the photon must be equal to the momentum exchanged with the box per time interval between two reflections at bottom and lid:

$$m_{eff}\,g = \frac{c}{L}\left(\frac{h\,\nu}{c_1} - \frac{h\,\nu}{c_2}\right) = \frac{h\,\nu}{c\,L}\,\Delta c \tag{6}$$

With (3) we obtain the dependence:

$$\frac{\Delta c}{c} = \frac{\Delta\Phi}{c^2} \tag{7}$$

so that the velocity of light is reduced closer to the gravitational center. This is in accordance with the observed deflection of light in gravitational lenses and reconciles the results of Pound and Alley. It is not necessary — and would actually not be true — to assume that the effective mass in (3) is attracted by a gravitational center in the same way as the rest mass of a particle. It is sufficient to assume that the momentum of photons increases at constant frequency due to a variation of the velocity of light when they approach a gravitational center.

IV The frequency of atomic clocks in a gravitational field

Having confirmed school I by Alley's experiment we are now confronted with equation (2) and must try to understand why the spacing of atomic levels depends on the gravitational potential. GRT says simply "time" runs faster at increased elevation, but this is an empty statement. As already said, it is not even true, if we identify physical time with the reading of a pendulum-clock or with the "day" as defined by the rotating earth. Having established a "world-time" which, in principle, is the reading of an atomic clock in Paris, we must concede that this time holds also on Mount Everest, not withstanding the fact that an atomic clock operating on its summit would run faster than that at Paris, Washington, or Braunschweig. It is easy enough to connect the two readings by formula (1) without entering a philosophical debate on the true nature of "time".

Quantum physics supplies us with a theory of the atomic energy levels. If we take the Rydberg energy

$$R_y\, c\, \nu = \frac{2\,\pi^2\, e^4}{h^2}\, m_e \tag{8}$$

as the basic energy determining the spacing of atomic levels, as is the case in hydrogen, we see that it depends on two constants of nature, both of which are independent of the gravitational potential to our knowledge (and to our liking). The electron mass is not constant according to experiment and to SRT, but depends on the velocity of the electron. It is not unlikely that it depends also on the gravitational potential, since energy is invested to lift an electron to a certain elevation. The obvious relationship would be:

$$c^2\, \Delta m_e = m_e\, \Delta\Phi \tag{9}$$

Substituting this into (8) would yield the frequency shift of equation (1) which was observed in Alley's experiment. Whether GRT is happy with assumption (9) remains to be seen. In any case, it is a more physical hypothesis than a speculation on mystic "properties of time", a *contradictio in adjecto* in view of Kant's profound analysis.