The history of displacement current

Further explanation of an earlier article

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As a result of correspondence following their article "Displacement current" in the December 1978 issue, the authors feel that further explanation of their views is required. They offer it in the form of this brief historical survey.

IN THE EARLY nineteenth century electromagnetic theory made advances, a cornerstone of the theory being the doctrine of conservation of charge q, which developed into the doctrine of continuity of electric current flow, $dq/dt = i$.

In the middle of that century Maxwell struggled with the paradox of the capacitor, where charge entered one plate and then flowed out of the other plate apparently without traversing the space between the plates (Fig. 1). It seemed that electric charge was being destroyed on the upper plate and being re-created when it reappeared on the lower plate. Maxwell "cut the Gordian knot" as Heaviside put it (Heaviside 1893) by postulating a new type of current, called "displacement current", as flowing across the gap BC in Fig. 1 so as to save the principle of continuity of electric current.

"Displacement current" was a result of his postulation of "electric displacement". Maxwell said that the total outward displacement across any closed surface is equal to the total charge inside the closed surface (Maxwell 1873).

It is not surprising that objections were raised. Notice, in Fig. 2, that if in any circuit there should be a break, BC, in the current path, we are bound by the principle of conservation of charge to say that the current i, that is the flow of charge, entering B from A accumulates as charge $\int idt$ at B, and the current reappearing at C "accumulates" as equal negative charge $-\int idt$. By definition, electric displacement outward from B equals the total charge trapped at B; $D = \int idt$ and $i = dD/dt$. It is not a coincidence that "displacement current" saves the idea of continuity of electric current; *it does so by definition.* With the postulation of displacement current, it would never in future be possible to devise an experiment which might refute the principle of continuity of electric current. Popper would therefore say that "displacement current" is an unscientific concept (Popper 1963). Whenever charge seems to disappear at a point, displacement takes its place. Whenever electric current seems to disappear at a point, displacement current takes its place.

It is important that Maxwell and Heaviside believed that the current entering a capacitor plate became trapped and had nowhere to go. Writers on the subject must be glad that some route between B and C for real current did not declare itself, since they say that the brilliant postulation of displacement current led to the postulation by Maxwell of waves in space.

Meanwhile, even as Maxwell was contemplating the ethereal displacement current, practical electricians were inventing and building wired telegraph systems. The distortion of signals travelling long distances was bad, and was thought to be due to the fact that the capacitance of the telegraph wires had to be charged up through the resistance of the wires, resulting in an RC time constant which attenuated different frequencies differently. As late as 1910 virtually all electricians (including Lord Kelvin) did not accept Oliver Heaviside's claim that a telegraph wire had distributed inductance as well as capacitance, and that if only this inductance were increased by the addition of periodic loading coils, distortion-free transmission over long distances could be achieved (Heaviside 1893).

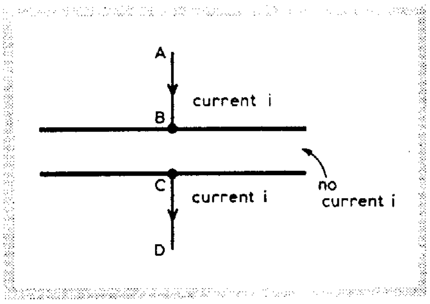

Fig. 1. *Charge flowing into one plate of a capacitor, as current i, and flowing out of the other plate.*

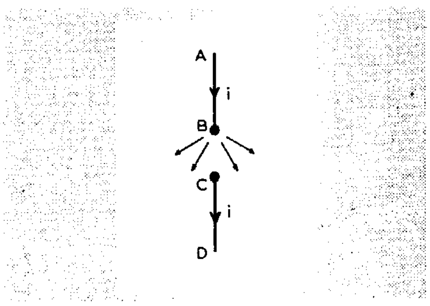

Fig. 2. *Electrical circuit AD with a break in the current path at BC. Charges accumulate at B and C.*

It was important for Heaviside to encourage a sensible approach to the characteristic impedance of telegraph lines, because the practical pay-off in telegraphy and telephony would be immense. (This misunderstanding delayed the introduction of telephones for twenty years.) This practical pay-off would be best achieved by arguing that signals travelling down (between) telegraph lines were undistorted TEM and similar to the waves in space discovered by Hertz in 1887, twenty years before, and previously postulated by Maxwell as one implication of his proposed displacement current.

It was important for Heaviside not to criticise the theory he was trying to argue from, Maxwell's electromagnetic theory. So it would have been injudicious for Heaviside to question the concept of displacement current, and he never did.

The essence of the concept of a transverse electromagnetic wave, TEM, is that nothing – field, flux, or current – flows laterally across the surface of the wave front. The analogy is the Severn Bore, where we see a single step of water rushing up the River Severn. Everything ahead of the step is steady, and everything behind the step is steady. There is no lateral, sideways flow. In the electromagnetic case (Fig. 3), the idea of a lateral flow of current across the face of a TEM step is absurd, and would result in a longitudinal magnetic field; the step would "get ahead of itself". Further, since the step travels forward at the speed of light, $1/\sqrt{\mu\epsilon}$ any lateral flow would cause embarrassment by travelling even faster, in the same way that when you walk across inside a moving train by Pythagoras' Theorem you are travelling faster than the train.

Now although in the case of a capacitor, displacement current needed to be regarded as just like a real current, for instance causing a magnetic field; in the case of the D flux at the front of a step of TEM ($E\times H$) energy current travelling down a telegraph line, the displacement clearly must not behave like a real current – for instance by creating a magnetic field which would reach out ahead of the wave front and ruin its TEM nature.

Maxwell and later Heaviside did not notice the discrepancy in the requirements of displacement current; that in a capacitor it must act like a real current but in a transmission line it must not; because neither of them knew that a capacitor is no more nor less than a transmission line (*Wireless World*, Dec. 1978, p. 51). This is even today known by very few scientists. Maxwell, along with today's text-book writers (e.g. Fewkes 1956, Bleaney 1957), believed that the displacement current dD/dt travelling across between the plates of a capacitor BC was uniformly distributed, and it is only very recently that it has been pointed out that the flow of current and field in a capacitor is identical with that in a transmission line; that the field moves out from the capacitor's leads as if they were links to one end of a transmission line. So the discrepancy could not become apparent.

A serious difficulty for displacement current arises when we realize that the two plates, BB′, CC′ in Fig. 4, *are* a transmission line. We know that the current i travelling down to B from A then flows out sideways from B along the capacitor plate BB′. This route, along the capacitor plates, failed to declare itself to Maxwell, and everyone has followed his lead.

In a transmission line (Fig. 4), everyone agrees that the current i entering the line at B leaves B by flowing along the line BB′. No displacement current dD/dt between the lines is needed for us to retain the doctrine of conservation of charge and conservation of current. In fact, if this dD/dt *were* regarded as a current, far from saving the doctrine, it would destroy it, because now more current ($i+dD/dt$) would be leaving the first section of the plate BB′ than was entering it. The last sentence is difficult to grasp; no matter, because it is easy to see, and sufficient to see, that if i enters B from A and i leaves B along BB′, continuity of current is preserved without our having to postulate displacement current.

"But surely we cannot just drop displacement current when for a century every expert (e.g. Solymar 1976, Winch 1963) has been protesting that it is the foundation of our craft; that 'Maxwell's leap of genius' in proposing displacement current was what got the subject going – leading to Hertz's discovery of waves in space, for instance?"

The answer lies hidden in Heaviside's magnificent, regal statement, "We reverse this." In his "Electrical Papers", Vol. 1, 1892, page 438, Heaviside wrote;

> Now, in Maxwell's theory there is the potential energy of the displacement produced in the dielectric parts by the electric force, and there is the kinetic or magnetic energy of the magnetic induction due to the magnetic force in all parts of the field, including the conducting parts. They are supposed to be set up by the current in the wire. We reverse this; the current in the wire is set up by the energy transmitted through the medium around it . . .

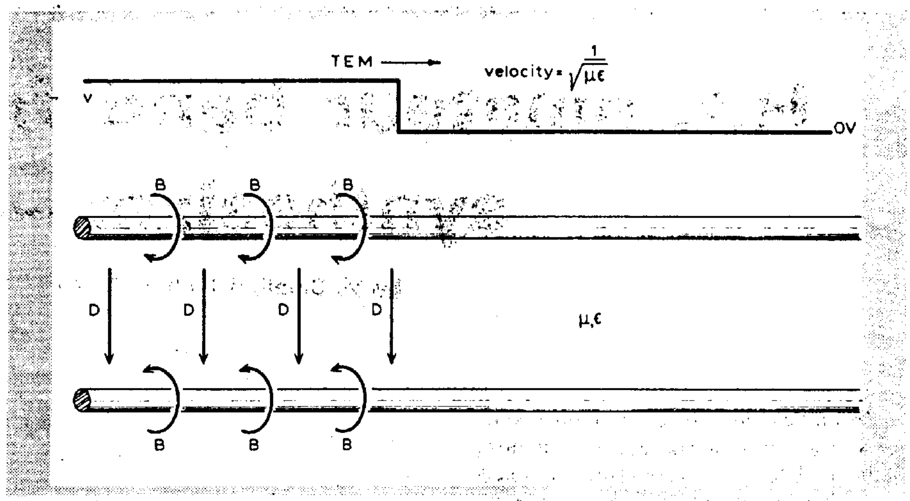

Fig. 3. *A TEM step (top) travelling at the speed of light and guided by two wires (below). The B arrows represent magnetic flux lines and the D arrows electric strain between the wires.*

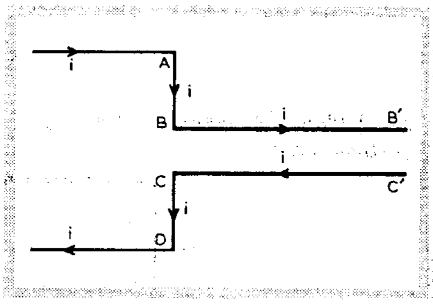

Fig. 4. *Current flowing into and out of capacitor plates BB′ and CC′. These two plates together constitute a transmission line.*

The discrediting of displacement current merely makes Heaviside's "We reverse this" mandatory. It means that the field must be the cause and electric current an effect, rather than (as Maxwell thought) the other way round.

If we keep to "Theory H", the theory that the field $E\times H$, travelling along between the wires at the speed of light – what Heaviside called the "energy current", is the cause, then electric charge and electric current are merely what define the *edge* of an energy current. If electric current is that which defines the side of an energy current, then we may with equal justification postulate "displacement current" as that which defines the front face of a step of energy current. Under "Theory H", Maxwell's 'leap of genius' (in postulating displacement current and thence waves in space) becomes tautological; "Because a wave in space if it existed would have to have a front face (displacement current), then I propose such a front face and therefore I propose waves in space."

Maxwell would have saved us a century of confusion if he had had enough insight to say, "Since circuits containing capacitors, that is, open circuits, work, it follows that the essence of electromagnetics cannot be electric current in closed circuits of conductors; it must be something else. What about waves in space?" Heaviside, seventy years ago, missed the key point by a whisker. He failed, but he failed gloriously. He never discovered the flaw in the structure, displacement current.

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Further reading

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Catt, I., "The rise and fall of bodies of knowledge". *The Information Scientist*, 12 (4), December 1978, pp. 137-144. □

The next seminar by the authors on digital electronics design will be held at St Albans on May 3-4. For information, contact C.A.M. Publishing, 17 King Harry Lane, St Albans, Herts.