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# Mathematics

Mathematics is a product of the human mind. It belongs to the realm of pure thought and has a life of its own which is not necessarily related to, or associated with, other disciplines or intellectual activities. It is usually pursued in a way similar to the writing of literary fiction, i.e. without any relevance or correspondence to real life. Of course, parts of mathematics have achieved significance in physical science, but it appears that this happened unintentionally, more by coincidence than by design.

Mathematics deals with concepts and codes which take their beginning in the imagination of mathematicians and which frequently remain inapplicable in practice. It differs from fictional literature insofar as its "language" uses symbolic, non-verbal means of expression, and its "grammar" is a system of abstract operational rules characterised by a high degree of consistency. This consistency and the precision inherent in quantitativeness are the pillars on which the strength of mathematics rests. The application of the rules produces consequences which follow from initial propositions with necessity and achieve a standard of exactitude not attainable by means of verbal language.

Mathematicians have erected an imposing theoretical edifice, but they have not been able to establish with any degree of certainty the foundations of their structure. They don't know the real nature of numerals and symbols, and the real meaning of their rules. Fortunately, it is the superstructure and not the foundations which has a practical significance in the study of phenomena that can be quantified, represented by symbols and correlated with mathematical rules of operation. This applies particularly to the study of the physical world. We find that mathematics has become a powerful tool in the hands of physicists, so powerful indeed that a separate group of theoretical physicists has developed who make it their business to expand the status and the influence of mathematics in physics.

When mathematical methods are used in physics, the boundary between the two disciplines is sometimes hard to discern. They appear to be so closely integrated that a distinction is considered difficult and unnecessary. However, in order to understand relativity it is important to delineate exactly in what way and to what extent mathematics is used legitimately in physics.

The first task is to identify a physical situation which contains measurable or quantifiable components. This task is usually performed by the physicist, and no mathematics is involved at this stage. The selection of suitable phenomena is a subjective process guided by the intellect of the individual researcher. Previously achieved results and conventions are taken into account, but can be disregarded if it is felt that they are not firmly established.

It is during the first phase that the limitations of mathematics become apparent. The establishment of a quantifiable component is associated with a considerable narrowing of the width and range of the available empirical and inductive material. The mathematical method is similar to a searchlight which illuminates a small and very circumscribed spot intensively, but leaves the immediate environment and the remainder of a vast domain in complete darkness. The physicist has to be selective, but the criteria for selection are by no means clear and his freedom is severely curtailed by the constraints of quantitativeness. Furthermore, the effectiveness and positive value of any mathematical operations depends on the soundness of judgment exercised in the preparation for the encoding of the physical components. It must be clearly specified what the quantities and symbols stand for, which is not always possible. Many difficulties arise during the translation of complex physical situations into available mathematical means of expression.

The transition from the first step to the second, which is represented by the mathematical manipulation, involves bridging a gap between the real, live and non-mathematical matrix of nature and the artificial, fragmented and restrictive symbolism of mathematics, and the question whether this bridging can be accomplished successfully has been asked in the past. The opinion was expressed that *mathematics cannot be applied with exactness to reality* (Boutroux). Einstein himself, contradicting what he said on other occasions, expressed doubt in his *Sidelights on Relativity* when he declared: *As far as the laws of mathematics refer to reality they are not certain.* We know that mathematics, at least to some extent, is applicable to the visible world as we perceive it, but that does not necessarily mean that physical phenomena can be, or are, described adequately by mathematical means.

The second stage is represented by the mathematical formalism. It begins with the encoded premise which is manipulated in accordance with the rules of the game in order to obtain mathematical consequences. During this stage mathematics is in complete control. Ideally, one should be able to discern the physical meaning of the mathematical operations which are being performed with the encoded information, but in actual fact we are relying on the well-established belief that the application of mathematical formalism at this stage is appropriate and adequate in terms of physical knowledge.

The third step in the process of using mathematical methods in physics is the decoding and the semantic and physical determination of the results. The consequences of the formalistic manipulation must be interpreted in physical language. Their practical meaning must be discovered if it is not immediately intelligible. They must be verified, and integrated into the existing body of knowledge. The third stage is basically non-mathematical and should be executed by physicists, not by mathematicians. If the mathematical results are physically unintelligible and do not represent anything occurring in nature, they must not be considered part of physics.

In the last fifty years or so the theoreticians have introduced a considerable and not always warranted complexity into mathematical structures associated with physical problems. As a result of this complexity difficulties have arisen in attaching physical meaning to mathematical consequences. This, in turn, has lent substance to the claim by the theoreticians that they are the sole arbiters and interpreters in the third phase of the process, and has led to the assertion that an interpretation in physical terms is not always necessary. Finally, the theoreticians have also taken over the first phase of the whole process and are using empirically insufficiently supported and purely hypothetical situations to derive data for the mathematical input. There is nothing wrong with the manipulation of hypothetical data as long as the whole process clearly retains its hypothetical status and remains within the realm of mathematics. But it is improper to obscure or to obliterate the dividing line between physical facts and mathematical imagination, and to introduce fictional elements into physical science.

The theoreticians reject the distinction between physics and mathematics. In their opinion mathematics is not just a method which can be applied in physics, but a source of knowledge, and even a more important source than the examination of nature. In Born's words *physics is deliberately and increasingly turning away from perceptibility as a source of knowledge and demanding criteria of greater acuity.* The theoreticians urge us to forget the proven methods of natural science and to embrace the new criteria of cognition based on mathematics. There is, of course, nothing new in this suggestion. It is a resurrection of the hollow and non-representational logic of mediaeval scholasticism in a different dress. The attempt is made to restore this particular metaphysical approach *to its old position of authority over experience* (Dingle).

The difference between physics and mathematics is that the former recreates our external environment while the latter reflects only our own thought processes which may or may not assist physics to perform its tasks. Physics recreates nature which has an independent existence and qualitative fullness, and is subject to a continuous process of discovery. The search for reality and truth is implicit in this process. Mathematics, on the other hand, has no independent existence, and is not anchored in reality. Although it possesses a high degree of internal consistency, it is not able to distinguish between right and wrong except in relation to its own rules. It is intellectually limited because it can deal only with quantitative phenomena. Axiologically it is indifferent, and spiritually unsatisfying.

Mathematics has been permitted to encroach on areas which are the proper domain of physics, and the encroachment has developed into what looks like absolute control over the whole domain. The door to this dominance was opened in 1873 by Maxwell's equations in electromagnetism, and in the 20th century supremacy was achieved and consolidated through the theory of relativity and quantum mechanics.

As long as the mathematical side of physics was represented by comparatively simple devices, the physicists had no difficulty in retaining control of the situation. But Maxwell's equations elevated mathematics to a higher level. Such a set of equations, assumed to be governing the behaviour of a whole subfield of physics, is a fruitful testing ground for mathematical experimenters. It presents a challenge to the theoretician as it is a ready-made axiomatic base from which deductions can be made. Unfortunately, during this process the formalistic treatment has the tendency to become detached from the physical meaning of the original axioms and to acquire an independent life. The mathematicians are eager to apply all possible instruments of their tool kit. They immerse themselves in the mathematical development of their ideas and are no longer interested in retaining contact with physical reality. If this game is pursued long enough and with determination, the contact with reality may be lost altogether.

When the relationship between physics and mathematics is discussed and the improper influence of mathematics noted, it must be made clear that there is absolutely no point in attacking the rules which govern the operation of mathematics in general or the application of these rules in a specific, mathematically encoded situation, even if the situation has no intelligible physical background or the encoding is controversial. The rules have in all cases their independent validity, and the mathematicians themselves will see to it that they are scrupulously adhered to. If the rules are violated, the mathematicians can be relied upon to discover the violation and not to rest until it has been rectified. The physicist need not worry unduly about the intermediate or purely mathematical stage of the three-phased process described above, although a certain amount of critical supervision may be necessary to ensure that parallelism between mathematical and physical operations is preserved where it can be recognised.

The mathematical treatment as such cannot change or remove the initial premises and assumptions which have been used for the definition and encoding of physical factors, and it is primarily the truth-value of these initial, non-mathematical, stated or unstated propositions which determines whether the mathematical consequences and their interpretation will be in accordance with physical reality, i.e. whether the resulting propositions will be true or false in terms of physics. The mathematical treatment is neutral. The consistency of the rules will ensure that a distortion of physical assumptions will not occur during the mathematical treatment.

The great advantage of the mathematical method, its consistency and reliability, is also its greatest weakness. If there is an error in the encoding of the premises, there will be an error in the conclusions. Mathematical manipulation will not remove or reduce it. On the contrary, the reliability of the mathematical method will tend to obscure any unreliability inherent in the premises, and a feeling of false security may be created.

Any attempt to find flaws in the mathematical operations associated with a physical theory is usually a complete waste of time as far as any non-mathematician is concerned. It is highly improbable that any real flaws could escape the attention of mathematicians. And it is not the mathematical formalism which is ultimately of importance, but the truth or falsity of the physical content of the theory and its metaphysical presuppositions. The mathematical apparatus of a theory may be as complex and as forbidding as the state of the art permits, semantic and logical intelligibility is still of incomparably greater significance. One does not need to have knowledge of higher mathematics to examine the crucial elements of a physical theory, and there is no justification in the belief that relativity can be understood only by means of mathematical formulas, that its propositions can only be stated and its conclusions can only be proved mathematically. This belief is cultivated by the theoreticians, and it reflects nothing but their witch-doctor mentality.

A theory which is dependent on mathematics to such degree that it cannot be understood by non-mathematicians belongs to the fictional structures of mathematics and has no place in physics. When mathematics is used not to elucidate physical problems, but to obscure them, its applicability must be questioned. When a physical theory requires a warning sign: *No entry for non-mathematicians* (Wien), it is highly probable that the theory has no common ground with physics. The reverse side of the warning sign should contain a complementary and equally categorical stop signal: No exit for mathematicians. What the mathematicians do within the confines of their warning signs is their business, but they should not gain admittance to the world of physical events and real things unless they promise to desist from using the tools of their craft as a strait-jacket.

The consistency of the mathematical formalism frequently serves as a basis for the assertion that the theory of relativity is endowed with a high degree of *inner consistency* and that this inner consistency is a sign of its strength. The relativists seem to forget that the inner consistency of a theory is not a sign of strength, but only a preliminary and necessary condition for it. Internal inconsistency would not reduce the strength of a theory, it would disqualify it. The real strength of a theory lies in its consistency with the external world.

Mathematics is important in physics because of its power to predict. With the help of mathematics complex motions of bodies can be analysed and antecedent and subsequent states of a physical situation reconstructed with great precision. Individual factors of a predetermined physical system can be exactly calculated. But no amount of mathematical manipulation will produce a new basic concept or a new fundamental insight in physics. The functional processes which lead to achievements of this kind are non-mathematical. They are based on semantics and verbal logic.