The Schwarzschild solution and its implications for gravitational waves

By

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“Schwarzschild’s Solution” **is not** Schwarzschild’s solution

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin\theta^2 d\varphi^2\right)$$

David Hilbert’s corruption of Schwarzschild’s solution, wherein it is claimed, **without proof**, that r can go down to zero, producing an **event horizon** at $r = 2m$ and an **infinitely dense** point-mass singularity at $r = 0$.

Schwarzschild's Solution

$$ds^2 = \left(1 - \frac{\alpha}{R}\right)dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2\left(d\theta^2 + \sin\theta^2 d\varphi^2\right)$$

$$R = R(r) = \left(r^3 + \alpha^3\right)^{1/3}, \qquad 0 < r < \infty.$$

There is no black hole.

3-D Spherically Symmetric Metric Manifolds

(Euclidean Space)

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin\theta^2 d\varphi^2\right)$$

$$0 \le r < \infty.$$

$$R_p = \int_0^r dr = r, \qquad A_p = 4\pi r^2, \qquad V_p = \frac{4}{3}\pi r^3$$

Non-Euclidean Metric
(Spherically Symmetric)

$$ds^2 = dR_p^2 + R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$
$$= \Psi\left(R_c\right)dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$R_c = R_c\left(r\right), \qquad R_c\left(0\right) \le R_c\left(r\right) < \infty.$$

$R_c(r)$ and $\Psi(R_c)$ are *a priori* **unknown** functions.

Non-Euclidean
Geometric Relations

$$R_p = \int_0^{R_p} dR_p = \int_{R_c(0)}^{R_c(r)} \sqrt{\Psi\left(R_c(r)\right)}\, dR_c(r) = \int_0^r \sqrt{\Psi\left(R_c(r)\right)} \frac{dR_c(r)}{dr} dr,$$

$$C_p = R_c(r) \int_0^{2\pi} d\varphi = 2\pi R_c(r), \qquad A_p = R_c^2(r) \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi = 4\pi R_c^2(r),$$

$$V_p = \int_0^{R_p} R_c^2(r) dR_p \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi = 4\pi \int_{R_c(0)}^{R_c(r)} \sqrt{\Psi\left(R_c(r)\right)}\, R_c^2(r) dR_c(r)$$

$$= 4\pi \int_0^r \sqrt{\Psi\left(R_c(r)\right)}\, R_c^2(r) \frac{dR_c(r)}{dr} dr$$

Spherically Symmetric Surfaces

$$ds^2 = r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

What is r ?

$$ds^2 = R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$R_c = R_c\,(r),$$

What is $R_c(r)$?

Gaussian Curvature (K)

$$K = \frac{R_{1212}}{g}$$

$$R_{\mu\nu\rho\sigma} = g_{\mu\gamma} R^{\gamma}_{.\nu\rho\sigma}$$

$$R^{1}_{.212} = \frac{\partial \Gamma^{1}_{22}}{\partial x^{1}} - \frac{\partial \Gamma^{1}_{21}}{\partial x^{2}} + \Gamma^{k}_{22}\Gamma^{1}_{k1} - \Gamma^{k}_{21}\Gamma^{1}_{k2}$$

$$\Gamma^{i}_{ij} = \Gamma^{i}_{ji} = \frac{\partial\left(\frac{1}{2}\ln\left|g_{ii}\right|\right)}{\partial x^{j}}$$

$$\Gamma^{i}_{jj} = -\frac{1}{2g_{ii}} \frac{\partial g_{jj}}{\partial x^{i}} \quad (i \neq j)$$

$$i, j, k = 1,2,3 \qquad x^{1} = \theta, \quad x^{2} = \varphi$$

$$\textit{all other } \Gamma^{i}_{jk} \textit{ vanish}$$

Gaussian Curvature (K)

(Spherically Symmetric Surface)

$$K = \frac{1}{R_c^2}$$

Gaussian curvature is an **intrinsic** geometrical property of a surface, completely independent of any embedding space (**Theorema Egregium** of C. F. Gauss). Gaussian curvature is a bending invariant. Its identity is **not altered** if the surface is embedded in a higher-dimensional space.

Standard Derivation

$$ds^2 = e^{\lambda} dt^2 - e^{\beta} dr^2 - R^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2 \right)$$

Generalisation of the Minkowski squared arc-length, where $R=R(r)$.

$$ds^2 = e^{\lambda} dt^2 - e^{\beta} dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2 \right)$$

Reduced generalisation of the Minkowski squared arc-length, where it is **asserted** that $R(r) = r$ and so that $R(0) = 0$, **without proof**.

For λ, β real valued functions of r alone, $e^{\lambda} > 0$ and $e^{\beta} > 0$ by construction: **e^{λ} and e^{β} cannot change sign or become zero.** The metric signature is thereby fixed at (+,-,-,-).

General Line-Element for *Ric* = 0

$$ds^2 = A(R_c)dt^2 - B(R_c)dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$A(R_c), B(R_c), R_c(r) > 0$$

General Solution for *Ric* = 0

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right)dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$R_c(0) \le R_c(r) < \infty$$

Proper Radius

$$R_p = \int \frac{dR_c}{\sqrt{1-\frac{\alpha}{R_c}}} = \sqrt{R_c\left(R_c-\alpha\right)} + \alpha \ln\left(\sqrt{R_c} + \sqrt{R_c-\alpha}\right) + k$$

For some r_o, $R_p(r_o) = 0$; hence $R_c(r_o) = \alpha$, $k = -\alpha \ln\sqrt{\alpha}$.

$$R_p\left(r\right) = \sqrt{R_c\left(R_c-\alpha\right)} + \alpha \ln\left(\frac{\sqrt{R_c} + \sqrt{R_c-\alpha}}{\sqrt{\alpha}}\right)$$

$$R_c = R_c\left(r\right)$$

Brillouin's Solution

$$ds^2 = \left(1 - \frac{\alpha}{r+\alpha}\right)dt^2 - \left(1 - \frac{\alpha}{r+\alpha}\right)^{-1} dr^2 - (r+\alpha)^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$0 < r < \infty$$

Droste's Solution

$$ds^2 = \left(1 - \frac{\alpha}{r}\right)dt^2 - \left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$\alpha < r < \infty$$

Admissible form for $R_c(r)$

$$R_c(r) = \left(\left|r - r_o\right|^n + \alpha^n\right)^{1/n} = \frac{1}{\sqrt{K(r)}}, \quad r \in \mathbf{R},\ n \in \mathbf{R}^+,\ r \neq r_o$$

Solution for $Ric \equiv R_{\mu\nu} = 0$

$$ds^2 = \left(1 - \frac{\alpha}{R_c}\right) dt^2 - \left(1 - \frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R_c^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$R_c(r) = \left(\left|r - r_o\right|^n + \alpha^n\right)^{1/n} = \frac{1}{\sqrt{K(r)}},$$

$r \in$ **R**, $n \in$ **R**$^+$, $r \neq r_o$

r_o and n entirely arbitrary constants

$R_c(r_o) = \alpha$ and $R_p(r_o) = 0$ are **scalar invariants**.

Isotropic Coordinates

$$ds^2 = \frac{\left(1-\frac{\alpha}{4h}\right)^2}{\left(1+\frac{\alpha}{4h}\right)^2} dt^2 - \left(1+\frac{\alpha}{4h}\right)^4 \left[dh^2 + h^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

$$h = h(r)\left[\left|r - r_o\right|^n + \left(\frac{\alpha}{4}\right)^n\right]^{1/n} \qquad R_c(r) = h(r)\left(1+\frac{\alpha}{4h(r)}\right)^2 = \frac{1}{\sqrt{K(r)}}$$

$$R_p(r) = h(r) + \frac{\alpha}{2}\ln\left(\frac{4h(r)}{\alpha}\right) - \frac{\alpha^2}{8h(r)} + \frac{\alpha}{4}$$

$$R_p(r_o) = 0, \quad R_c(r_o) = \alpha, \quad K(r_o) = \alpha^{-2} \quad \forall r_o \ \forall n$$

$$r \in \mathbf{R}, \quad n \in \mathbf{R}^+, \quad r \neq r_o$$

Kerr-Newman Geometry

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta\, d\varphi^2\right)^2 - \frac{\sin^2\theta}{\rho^2}\left[\left(R_c^2 + a^2\right)d\varphi - a\,dt\right]^2 - \frac{\rho^2}{\Delta}dR_c^2 - \rho^2 d\theta^2$$

$$R = R(r) = \left(\left|r - r_o\right|^n + \beta^n\right)^{1/n}, \qquad \beta = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - \left(q^2 + a^2\cos^2\theta\right)},$$

$$\rho^2 = R_c^2 + a^2\cos^2\theta, \qquad \Delta = R_c^2 - \alpha R_c + q^2 + a^2,$$

$$a = \frac{2L}{\alpha}, \qquad q^2 + a^2 < \frac{\alpha^2}{4},$$

$$r \in \mathbf{R}, \quad n \in \mathbf{R}^+, \; r \neq r_o$$

n and r_o arbitrary constants; $R(r_o) = \beta$ is a **scalar invariant**.

There is no black hole.

Newtonian Approximation

$$ds^2 = \left(c^2 - \frac{2m}{r} \right) dt^2 - \left(1 - \frac{2m}{rc^2} \right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin\theta^2 d\varphi^2 \right)$$

Claimed to be a one-body configuration.

But escape velocity

$$v = \sqrt{\frac{2Gm}{r}}$$

is a two-body concept.

Newton's Gravitation

$$F_g = -G\frac{mM}{r^2}$$

is a two-body theory.

$$\Phi = -\int_{\infty}^{r} \frac{F_g}{m}\, dr = -G\frac{M}{r}$$

Newtonian potential is a two-body concept.

$$P = m\Phi = -G\frac{mM}{r}$$

Newton's potential energy involves two bodies.

Special Relativity

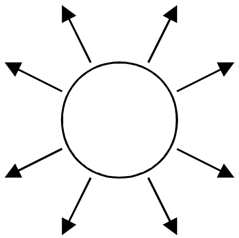
$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad V = L_o^3 \sqrt{1 - \frac{v^2}{c^2}}$$

$$D = \frac{m}{V} = \frac{m_o}{L_o^3 \left(1 - \frac{v^2}{c^2}\right)} \qquad v \to c \Rightarrow D \to \infty$$

Infinite density is forbidden.

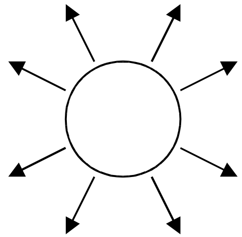
Black Hole Interactions

SBH ≡ **S**chwarzschild **B**lack **H**ole



SBH_1

Ric = 0: no matter present out here



SBH_2

SBH_1 is in the **matter-less** spacetime of SBH_2 and SBH_2 is in the **matter-less** spacetime of SBH_1. Thus the two black holes mutually persist in and mutually interact in a mutual spacetime that **by construction contains no matter**! Contra hyp!

$Ric = R_{\mu\nu} = 0$ is **<u>not</u>** a two-body configuration.

The Principle of Superposition **<u>does not apply</u>**.

Riemann Tensor Scalar Curvature Invariant (Kretschmann Scalar)

$$f = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$$

$$f = 12\alpha^2 K^3 = \frac{12\alpha^2}{R_c^6} = \frac{12\alpha^2}{\left(\left|r - r_o\right|^n + \alpha^n\right)^{6/n}}$$

$$f\left(r_o\right) = \frac{12}{\alpha^4} \quad \forall r_o \, \forall n$$

$$R_p\left(r_o\right) = 0, \quad R_c\left(r_o\right) = \alpha, \quad K\left(r_o\right) = \alpha^{-2}$$

Radial Geodesic Acceleration of a Point

$$a = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)\left|g_{00,1}\right|}{2g_{00}}$$

Doughty's General Expression

$$a = \frac{\alpha}{R_c^{3/2}(r)\sqrt{R_c(r)-\alpha}}$$

$$\lim_{r \to r_o^{\pm}} R_c(r) = \alpha$$

$$r \to r_o^{\pm} \Rightarrow a \to \infty \quad \forall r_o \ \forall n$$

$$a = \frac{2m}{r^{3/2}\sqrt{r-2m}}$$

At $r = 2m$ the acceleration is infinite, where it is claimed by the astrophysical scientists that **there is no matter**!

When 0 < r < 2m in the Standard Metric

$$g_{00} = \left(1 - \frac{2m}{r}\right)$$

is negative

$$g_{11} = -\left(1 - \frac{2m}{r}\right)^{-1}$$

is positive

The signature changes from (+,-,-,-) to (-,+,-,-);

so the rôles of t and r are **interchanged**.

To amplify this set $t = r^*$, $r = t^*$.

Non-Static Solution to a Static Problem

$$ds^2 = \left(1 - \frac{2m}{t^*}\right) dr^{*2} - \left(1 - \frac{2m}{t^*}\right)^{-1} dt^{*2} - t^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$0 < t^* < 2m$$

The signature is (-,+,-,-).

This metric is a function of the timelike t^*: a time-dependent metric. **Contra hyp**!

Principle of Equivalence

According to Einstein, his ‘Principle of Equivalence’ and his laws of Special Relativity **must manifest** in a sufficiently small region of his gravitational field.

Both the ‘Principle of Equivalence’ and the laws of Special Relativity are defined in terms of **multiple** arbitrarily large finite masses.

But $Ric = 0$ is a spacetime that by construction **contains no matter**! Thus $Ric = 0$ violates the physical basis of Einstein’s gravitational field.

Empty Spacetimes

$$R_{\mu\nu} = \lambda g_{\mu\nu}$$

• λ *is* the 'cosmological constant'.

• The spacetime contains no matter by construction ($T_{\mu\nu} = 0$).

• Is satisfied by de Sitter's **empty** universe (which can be obtained by setting m = 0 in the so-called 'Schwarzschild/de Sitter metric'.

$$R_{\mu\nu} = 0$$

• The spacetime contains no matter by construction ($T_{\mu\nu} = 0$).

• Alleged to contain matter (***post hoc***), denoted by m in the so-called "Schwarzschild solution".

Thus $T_{\mu\nu}$ = 0 is alleged to both support material cause and to preclude material cause.

Empty Spacetimes

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\lambda}{3} r^2\right) dt^2 - \left(1 - \frac{2m}{r} - \frac{\lambda}{3} r^2\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

The "Schwarzschild – de Sitter" metric: $T_{\mu\nu} = 0$,

but said to contain material cause, *m*.

$$ds^2 = \left(1 - \frac{\lambda}{3} r^2\right) dt^2 - \left(1 - \frac{\lambda}{3} r^2\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

de Sitter's completely empty world: $T_{\mu\nu} = 0$,

said to contain **no** material cause.

Einstein's Field Equations

$$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0$$

- $G_{\mu\nu}/\kappa$ are the components of a **gravitational energy tensor**.
- The total energy of the gravitational field is **always 0**.
- The $G_{\mu\nu}/\kappa$ and the $T_{\mu\nu}$ **must vanish identically**.
- Gravitational energy **cannot be localised**.
- Einstein's gravitational field **violates** the usual conservation of energy and momentum established by experiment.

Einstein’s pseudo-tensor

$$\sqrt{-g}\, t_\nu^\mu = \frac{1}{2}\left(\delta_\nu^\mu L - \frac{\partial L}{\partial g^{\sigma\rho}_{,\mu}} g^{\sigma\rho}_{,\nu} \right)$$

$$L = -g^{\alpha\beta}\left(\Gamma^\gamma_{\alpha\kappa}\Gamma^\kappa_{\beta\gamma} - \Gamma^\gamma_{\alpha\beta}\Gamma^\kappa_{\lambda\kappa} \right)$$

definition of Einstein’s pseudo-tensor

Einstein's pseudo-tensor is meaningless

$$\sqrt{-g}\, t_{\mu}^{\mu} = \frac{1}{2}\left(4L - \frac{\partial L}{\partial g^{\sigma\rho}_{\ ,\mu}} g^{\sigma\rho}_{\ ,\mu} \right)$$

$$\frac{\partial L}{\partial g^{\sigma\rho}_{\ ,\mu}} g^{\sigma\rho}_{\ ,\mu} = 2L$$

So the invariant is L.

But such invariants do not exist!

Linearisation and Perturbation

Linearisation of Einstein's field equations is inadmissible because linearisation implies the existence of a tensor which, except for the trivial case of being precisely zero, **does not exist**!

(proven by the German mathematician Hermann Weyl, in 1944)

Conclusions

- “Schwarzschild’s solution” **is not** Schwarzschild’s solution. Black holes **are not** predicated by Schwarzschild’s solution.

- The quantity *r* in “Schwarzschild’s solution” **is the inverse square root of the Gaussian curvature of a spherically symmetric geodesic surface in the spatial section** and is therefore **not** itself a distance of any kind in the manifold.

- *Ric* = 0 is a spacetime that by construction contains no matter and therefore **violates** the physical principles of General Relativity (the ‘Principle of Equivalence’ and the laws of Special Relativity **cannot manifest** in a spacetime that **by construction contains no matter**).

Conclusions

•General Relativity **does not** predict black holes. **Black holes do not exist**.

•Einstein's gravitational field **violates** the usual conservation of energy and momentum established by experiment. **Einstein gravitational waves do not exist**.

•Despite the numerous claims of the astrophysical scientists, no black holes and no Einstein gravitational waves have been detected. Gravity Probe B **did not detect** the Lense-Thirring effect. The international efforts to detect black holes and Einstein gravitational waves **are destined to detect nothing**.

Acknowledgements

I thank **Monika Vandory** for providing the financial means for me to attend this conference. Without her generous benefaction I would not have been able to be here.

Dedication

For my late brother,

Paul Raymond Crothers

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